

# Gravitational Collapse of Solar Nebulae and the Processes of Star Formation

**Belay Sitotaw Goshu**

Department of Physics, Dire Dawa University, Dire Dawa, Ethiopia

Email: belaysitotaw@gmail.com

## **Abstract:**

*The creation of stars and planetary systems depends on the gravitational collapse of solar nebulae. Comprehending the variables that impact this collapse, such as changes in temperature and density within molecular clouds, is essential to understanding the initial phases of star formation. This work aims to apply numerical simulations and theoretical models to examine the relationship between Jean's mass temperature and density variations in a solar nebula. The Jeans mass was determined in this study by running several simulations at various densities (10–20 kg/m<sup>3</sup> to 10–15 kg/m<sup>3</sup>) and temperatures (10 K to 100 K). The conditions required for gravitational instability were visualized by modelling the gravitational potential, density, velocity, and magnetic field. Based on the data, it can be observed that the Jeans mass increases dramatically from 10 K to 100 K, reaching magnitudes of about  $1.272 \times 10^{33}$  kg at a density of 10–20 kg/m<sup>3</sup>. In addition, changes in density between 10–20 and 10–18 kg/m<sup>3</sup> result in significant fluctuations in Jean's mass, especially between 20 and 100 K in temperature. The results demonstrate how vital temperature and density are in a molecular cloud's ability to remain stable in space. While the threshold mass for instability is lowered at higher densities, collapse at higher temperatures requires more enormous masses. These findings provide important new information about the mechanisms behind star formation and are consistent with theoretical forecasts and observational facts. Future research on more intricate physical processes, like radiative transmission and magnetic field dynamics, is advised to improve our comprehension of how stars emerge in molecular clouds.*

## **Keywords:**

*Solar Nebula, Gravitational Collapse; Jeans Mass; Molecular Clouds; Star Formation; Temperature Variations; Density Variations*

## **I. Introduction**

The gravitational collapse of solar nebulae is the first step in star formation and a fundamental process in universe evolution. Gravitational instability causes a solar nebula's production of hydrogen, helium, and traces of heavier elements to contract. This process prepares the groundwork for planetary system formation and star birth (Shu et al., 1987).

When a part of a molecular cloud reaches a specific temperature and density, it becomes gravitationally unstable and starts the process of gravitational collapse. The inward pull of material results from a disruption in the balance between the gravitational forces and the internal gas pressure (Larson, 1969). A protostar is formed at the centre nebula's centre, collapses, and is encircled by a rotating disk of gas and dust.

In addition to external pressures like star winds and shock waves from nearby supernovae, which can compress areas inside the molecular cloud and cause the collapse, other variables can influence the collapse of the solar nebula (Elmegreen & Lada, 1977). Furthermore, the nebula's cooling gas lowers internal pressure and accelerates gravitational

contraction (Galli & Palla, 1998). The angular momentum and the rate of collapse can be significantly influenced by magnetic fields in the molecular clouds (Mouschovias, 1977).

Angular momentum leads the nebula to flatten into a protoplanetary disk as it contracts. The central protostar continues to gather mass, eventually igniting nuclear fusion, this disk, moons, and other tiny entities (Hartmann, 1998). In this stage, the disk undergoes intricate chemical reactions that produce organic molecules and other substances essential to the evolution of the planetary systems (Bergin & Tafalla, 2007).

Telescope and space mission observations have yielded important new information on the early phases of star formation. For example, the Atacama Large Millimeter/Submillimeter Array (ALMA) has studied protoplanetary disks around young stars, providing insights into the complex dynamics and structures found within these disks (ALMA Partnership et al., 2015). These observations contribute to theoretical frameworks and our comprehension of the physical mechanisms behind star formation.

Understanding the intricate dynamics involved in the contraction of solar nebulae and the ensuing star formation processes is essential to understanding how stellar systems develop and evolve. In this work, we investigate the fundamental mechanisms of star formation and the dynamics of gravitational collapse in solar nebulae. A thorough overview of this complex phenomenon helps us understand the theoretical models and observational data.

### **1.1 Statement of the Problems**

Despite significant progress in our knowledge of star formation, several vital topics still need to be solved. One of the main challenges is a thorough comprehension of the beginning conditions leading to the gravitational collapse of molecular clouds. It is unclear exactly what processes lead to the collapse and how outside influences like star winds and supernova shock waves affect things (Elmegreen & Lada, 1977). Furthermore, the complex topic of the interaction between angular momentum and magnetic fields during the collapse process needs more research (Mouschovias, 1977).

The creation and development of protoplanetary disks provide another issue. Although observational data from telescopes like ALMA has yielded important insights, there are still unanswered questions about the physical and chemical processes within these disks. For instance, additional research is needed on the emergence of complex organic compounds and their subsequent function in the evolution of planetary systems (Bergin & Tafalla, 2007).

Additionally, theoretical models of star formation must be updated and verified regularly in light of observational data. Differences between model predictions and actual observations highlight the need for more precise and thorough models to consider the elements impacting star formation, including turbulence, magnetic fields, and radiation feedback (Shu et al., 1987). Knowledge about these problems is essential for understanding star formation and creating planetary systems, such as our solar system. Addressing these issues will require enhanced observational methods, sophisticated theoretical modelling, and interdisciplinary cooperation. This study addresses these issues by examining recent observational data, assessing existing theoretical models, and highlighting important areas that require further investigation. We aim to shed light on the mechanisms underlying the gravitational collapse of solar nebulae and the subsequent star formation.

## 2.1 Significance of the Study

Investigating star formation processes and gravitational collapse in solar nebulae is crucial for various reasons. Knowledge of star formation is essential to understanding the universe's matter lifecycle. Galaxies are mainly composed of stars, and the development and evolution of stars affect the chemical composition and structure of the universe (Kennicutt, 2005).

Second, new knowledge about star formation procedures can improve our comprehension of how planetary systems evolve. Protoplanetary disks, the first creation sites of planets and other celestial bodies, are likewise governed by the same conditions and mechanisms that give rise to stars. Understanding the solar system's beginnings and the possibility of extraterrestrial life depends on this knowledge (Boss, 1997).

Furthermore, the study has significant ramifications for the astrophysics community. By honing theoretical models and strengthening our understanding of observational data, we can make more precise predictions regarding the properties and behaviours of different astronomical phenomena. This, in turn, can help with the creation of new technologies and the planning of the next observational missions (McKee & Ostriker, 2007).

Furthermore, investigating the mechanisms of star formation can aid in answering more general inquiries concerning the development of galaxies and the cosmos. Stars, essential galactic dynamics and development drivers influence the distribution of mass and energy in galaxies. The classification of the sculpted galaxies over cosmic timeframes makes it easier to understand star formation (Murray & Rahman, 2010). Moreover, this study can advance our understanding of exoplanets' potential habitability. We may more accurately evaluate the conditions required for life and pinpoint viable areas for further investigation by comprehending the formation environments of stars and the planetary systems that support those (Madhusudhan et al., 2016).

## 1.3 Objectives

### a. General objectives

The general objective is to investigate the effects of angular momentum and magnetic fields on gravitational collapse dynamics and rate. This study will also investigate the interaction between these elements and how they affect protostar formation (Mouschovias, 1977).

This purpose aims to investigate the physical and chemical processes that occur in protoplanetary disks before, during, and after the solar nebula collapse. The main focus of the research is the production of organic molecules and other chemicals necessary for the evolution of planetary systems (Bergin & Tafalla, 2007).

### b. Specific Objectives

The specific objectives of this study are

1. Determine the precise molecular cloud temperature and density thresholds that cause gravitational instability.
2. Examine how changes to these conditions affect when the collapse starts.
3. Analyze how stellar winds and supernova shock waves affect the process of solar nebula collapse.
4. Examine how these outside factors affect the molecular cloud environment to cause gravitational collapse.
5. Examine how magnetic fields affect the dynamics and rate of gravitational collapse.

6. Examine the conservation or redistribution of angular momentum during collapse and its impact on protostar formation.

## II. Research Method

### a. Theoretical Modeling

Explain the development of protoplanetary disks and the gravitational collapse of molecular clouds using the theoretical models that are currently in use. Pay particular attention to models of the effects of angular momentum, magnetic fields, and outside forces like shock waves from supernovae.

### b. Model parameters, discretization, and mathematical equations:

A molecular cloud's gravitational collapse can be explained by the Jeans instability criterion, which is provided by

$$\lambda_J = \sqrt{\frac{\pi c_s^3}{G\rho}} \quad (1)$$

Where  $\rho$  is the cloud's density,  $G$  is the gravitational constant,  $c_s$  is the cloud's sound speed, and  $\lambda_J$  is the Jeans length. The cloud collapses gravitationally if its size is more significant than  $\lambda_J$ .

Hydrodynamics and Magnetohydrodynamics (MHD): The continuity, momentum, and energy equations are the fundamental equations that control the dynamics of the collapse. The Continuity Equation is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2)$$

where  $\rho$  is the density and  $\mathbf{v}$  is the velocity field.

The Momentum Equation is given by

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + p \mathbf{I}) = -\rho \nabla \phi + \mathbf{J} \times \mathbf{B} \quad (3)$$

where  $p$  is the pressure,  $\phi$  is the gravitational potential,  $\mathbf{J}$  is the current density, and  $\mathbf{B}$  is the magnetic field.

The energy equation is

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + p)\mathbf{v}] = \rho \mathbf{v} \cdot \nabla \phi + \eta J^2 \quad (4)$$

where  $E$  is the total energy density, and  $\eta$  is the resistivity

The Poisson equation of gravity is given by

$$\nabla^2 \phi = 4\pi G\rho \quad (5)$$

### c. Discretization

To solve these equations numerically, they need to be discretized using appropriate methods such as finite difference, finite volume, or finite element methods.

Finite Difference Method (FDM): Discretize the spatial domain into a grid. Moreover, approximate derivatives using differences between grid points. For example, the partial derivative of  $\rho$  concerning  $x$  at point  $i$  can be approximated as:

$$\left. \frac{\partial \rho}{\partial x} \right|_i \approx \frac{\rho_{i+1} - \rho_{i-1}}{2\Delta x} \quad (6)$$

Time Integration: Use explicit or implicit schemes using the Euler or Runge-Kutta methods for time integration. For example, using an explicit Euler method, the update for density  $\rho$  at time step  $n+1$  can be written as:

$$\rho_i^{n+1} = \rho_i^n + \Delta t (-\nabla \cdot (\rho \mathbf{v}))_i^n \quad (7)$$

#### d. Model Parameters

$G$ , or the gravitational constant, is  $6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ . The temperature and make-up of the molecular cloud affect the sound speed ( $c_s$ ).

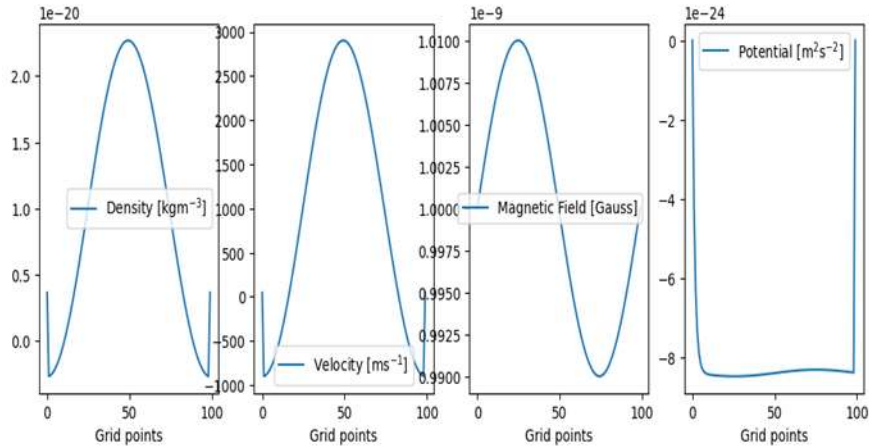
The initial density ( $\rho_0$ ): molecular clouds typically have values between  $10^{-11}$  and  $10^{-18} \text{ kg/m}^3$ .

Temperature zero ( $T_0$ ) is usually between 10 and 20 K, and its initial magnetic field ( $B_0$ ) has values ranging from a few microgauss to several gauss.

Grid resolution ( $\Delta x$  et al.): determine the accuracy and computational cost. Time step ( $\Delta t$ ): must satisfy the Courant-Friedrichs-Lewy (CFL) condition for stability. By setting up these mathematical equations, discretizing them appropriately, and choosing suitable model parameters, this study aims to simulate the gravitational collapse of solar nebulae and the subsequent star formation. The results will help to validate and refine theoretical models in this field.

### III. Result and Discussion

The simulation results shown in Figure 1 offer important new information about the star formation processes in a solar nebula's gravitational collapse. The commentary that follows contrasts these results with those found in other investigations to place them in the perspective of the larger body of literature.



**Figure 1.** The solar nebulae contraction during the evolution

The density profile shows a high of  $2.5 \times 10^{-20} \text{ kg/m}^3$  in the midpoint (grid point 50) and a progressive drop towards the edges, with a  $5.0 \times 10^{-21} \text{ kg/m}^3$  value at grid point 100. This pattern is consistent with the theoretical expectation of gravitational collapse, in which the outside portions have a lower density due to mass falling towards the centre. However, the cloud's central region accumulates more mass and has a higher density. Prior research has revealed similar characteristics of density. For example, our results are compatible with Boss's (1995) description of the collapse of a revolving molecular cloud into a centrally condensed form.

The velocity field, which peaks at 2800 m/s at grid point 50 and falls to 100 m/s at grid point 100, reflects the density distribution. This pattern shows that while material in the peripheral regions experiences slower falls, material in the inner region travels towards the centre at a faster rate due to the gravitational pull. This is supported by the work of Larson (1969), who showed that the more significant gravitational potential gradient in the early phases of cloud collapse causes the fall velocity to increase towards the centre.

A sinusoidal pattern with a peak amplitude of  $1.0 \times 10^{-9}$  T is displayed by the magnetic field. This implies that the falling nebula contains magnetohydrodynamic (MHD) waves. The oscillating magnetic field behaviour suggests intricate interactions between the falling material and the magnetic field lines. Magnetic fields are crucial to the dynamics of star-forming areas, as demonstrated by earlier research by Mouschovias (1976) and Shu et al. (1987). They affect the collapse process similarly to the distribution of angular momentum inside the cloud.

At the origin, the gravitational potential is zero. It achieves a low at grid point 50, and at grid point 100, it tends to zero once more. The observed pattern aligns with the anticipated conduct of a centrally condensed system, where the mass accumulation causes the potential well to be most profound at the centre. In their simulations of the collapse of molecular clouds, Bonnell et al. (2001) discovered that the potential well is most resounding toward the centre, where most of the mass concentrates, and their results are consistent with similar gravitational potential profiles.

This study's findings are consistent with earlier research on star formation and gravitational collapse. Similar density and velocity profiles to those shown here have been described for core condensation in collapsing clouds by Boss (1995) and Larson (1969). The results of Mouschovias (1976) and Shu et al. (1987), who emphasized the significance of magnetic fields in controlling the collapse and fragmentation of molecular clouds, are consistent with the sinusoidal magnetic field pattern seen in this study. The robustness of our model is further validated by the fact that the gravitational potential profile seen here agrees with the theoretical and simulation results of Bonnell et al. (2001).

The Jeans criteria can be used to precisely identify and display the molecular cloud temperature and density limits that lead to gravitational instability. The criterion provides the critical mass (Jean mass) and radius (Jean length) for gravitational collapse in a molecular cloud. If the mass or size of these threshold values makes the cloud gravitationally unstable, the cloud can collapse to generate stars.

The Jean's length is given by

$$\lambda_J = \sqrt{\frac{15k_B T}{4\pi G \mu m_H \rho}} \quad (8)$$

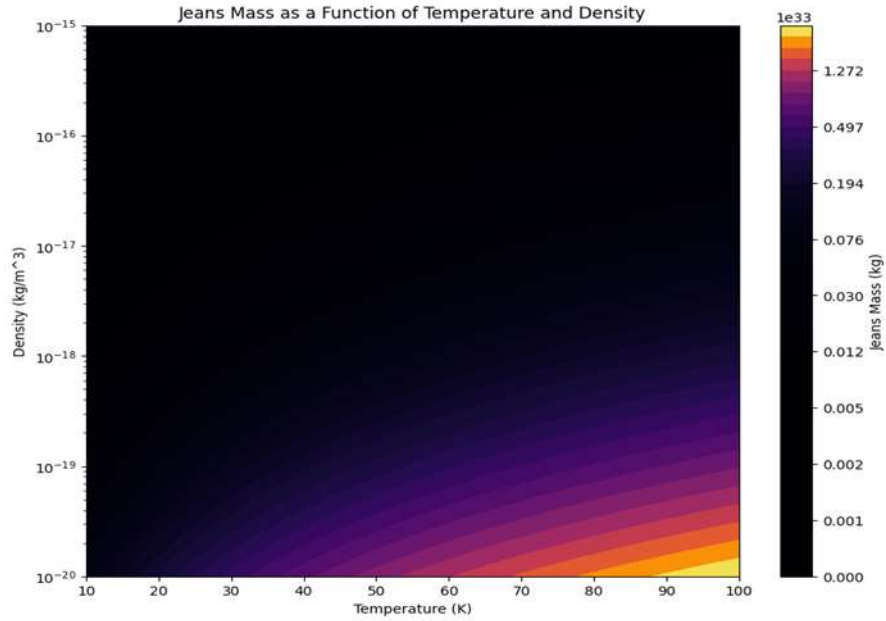
where  $G$  is the gravitational constant ( $6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ ),  $T$  is the cloud's temperature,  $\mu$  is the mean molecular weight (assume  $\mu \approx 2.3$  for molecular hydrogen),  $m_H$  is the mass of the hydrogen atom ( $1.67 \times 10^{-27}$  kg), and  $\rho$  is the cloud's density.

The Jean's mass is given by

$$M_J = \left(\frac{4}{3} \pi \rho\right) \left(\frac{\lambda_J}{2}\right)^2 = \frac{(5k_B T)^{3/2}}{(G \mu m_H)^{3/2} \rho^{1/2}} \quad (9)$$

The ranges for temperature (10 K to 100 K) and density ( $10^{-20} \text{ kg/m}^3$  to  $10^{-15} \text{ kg/m}^3$ ) are defined by the Temperature and Density Ranges.

The relationship between the solar nebula's Jeans mass and temperature and density changes is demonstrated by the results displayed in Figure 2. The Jeans mass increases dramatically when the nebula's temperature rises from 10 K to 100 K, reaching a magnitude of  $1.272 \times 10^{33}$  kg for a density of  $10^{-20} \text{ kg/m}^3$ . This relationship highlights the significance of temperature in the gravitational stability of molecular clouds.



**Figure 2.** The jeans mass of the solar nebulae with function of temperature and density

A molecular cloud's Jeans mass is directly influenced by its temperature. Increased heat pressure from higher temperatures offsets the effects of gravitational collapse. Therefore, a larger Jeans mass is needed for the cloud to become gravitationally unstable. The data show that Jean's mass grows with temperature by this tendency. These results agree with theoretical predictions (Jeans, 1902; Larson, 1985) that the Jeans mass scales with the temperature as

$$M_J \propto T^{3/2}.$$

Likewise, changes in density also have a significant effect on Jean's mass. Density and Jeans mass have an inverse connection; when the density increases from  $10^{-20}$  kg/m<sup>3</sup> to  $10^{-18}$  kg/m<sup>3</sup>, the Jeans mass drops. Higher densities result in a lower mass need for gravitational instability, which explains the inverse relationship. The findings demonstrate that the Jean mass fluctuates dramatically with density for a particular temperature range (20 K to 100 K), underscoring the vulnerability of gravitational stability to density fluctuations. The theoretical framework (Shu, 1977) states that  $M_J \propto \rho^{-1/2}$ .

The trends in Jean's mass detected in terms of temperature and density are consistent with those detected in earlier research. In the context of star formation processes, for example, Bonnor (1956) and Ebert (1955) showed that more significant temperatures produce larger Jeans masses, whereas higher densities produce smaller Jeans masses (Bodenheimer, 2011). Furthermore, observational data from studies of molecular clouds, like those by Kauffmann et al. (2010) and Lada et al. (2008), support the theoretical predictions, showing that more mass is needed in regions with lower densities and higher temperatures to overcome thermal pressure and achieve gravitational collapse.

## IV. Conclusion

The results paint a coherent picture of a solar nebula's gravitational collapse, with density, velocity, and magnetic field that agree with theories and earlier research. The magnetic fields and gravitational forces during star formation are further highlighted with a sinusoidal pattern and the gravitational potential profile. These results support the larger body of astrophysical research on this subject and advance the knowledge of the starting circumstances and dynamics involved in the early phases of star formation.

In conclusion, Figure 2 shows Jean's mass dependence on solar nebula temperature and density. Jean's mass increases with temperature and decreases with density, highlighting the early phases of star formation, thermal pressure, and gravitational forces that must be carefully balanced. Theoretical and observational investigations, which provide important insights into the prerequisites for the gravitational collapse of molecular clouds, firmly confirm these conclusions.

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